

# CS 4476(5413) MID TERM EXAMINATION

February, 2005

Duration: One hour

Name:

Student Number:

**Note:** Each student needs to solve 6 problems. Every student should solve problems in Set A. For students of CS 4476, choose 3 problems in Set B and 1 problem in Set C. For students of CS 5413, choose 2 problems from Set B and 2 problems from Set C. If you solved more than 6 problems, please indicate which of the problems you want to be marked.

## Set A

### Problem 1.

Do you agree the following statements? Give brief reasons to support your opinion.

1. A public key encryption system is more secure than a secret key encryption system, but the former system is not as efficient as the later one.
2. The main purpose of PKI is to manage public key certificates.
3. A monoalphabetic cryptosystem is not secure, because probabilistic methods may break the system even under a cipher text only attack.
4. The main methods for checking whether a hash function is secure are examining whether the hash function is a one-way function (i.e., it is difficult to find the inverse of the function).

**Problem 2.**

Let  $\mathcal{P} = \mathcal{C} = \mathbb{Z}_{26}$ , key space  $\mathcal{K} = \{(a, b) : a \in \mathbb{Z}_{26}^*, b \in \mathbb{Z}_{26}\}$ . Define the encryption function as

$$e_{(a,b)}(x) \equiv ax + b \pmod{26}.$$

Suppose  $(a, b) = (9, 5)$  and the plaintext is **good**. What is the ciphertext? The correspondences between  $\mathbb{Z}_{26}$  and English characters are as follows.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>
0	1	2	3	4	5	6	7	8	9	10	11	12
<hr/>												
<i>n</i>	<i>o</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>	<i>u</i>	<i>v</i>	<i>w</i>	<i>x</i>	<i>y</i>	<i>z</i>
13	14	15	16	17	18	19	20	21	22	23	24	25

## Set B

### Problem 3.

One encryption method is as follows. Let  $m$  be an integer. Write the plaintext in a table with rows of length  $m$ . The ciphertext is obtained by taking the columns of the table. For example, when  $m = 3$  and the plaintext is **networksecurity**, the table is:

```
net
wor
kse
cur
ity
```

So the ciphertext is **nwkcieosuttrery**. This is a special case of Permutation Cipher. Decrypt the ciphertext **imnrnpsinasiduetfttetctcyoicyoeuorouamrro**. (Note here  $m$  is not necessary 3).

**Problem 4.**

Suppose Alice and Bob only have an AES encryption algorithm  $e_K()$  and share a secret key  $K$ . Design a method (a mode) for them, which can be used for both encryption and decryption.

**Problem 5.**

Suppose Alice has an RSA system with public key  $(n_1, b_1)$  and private key  $(a_1, p_1, q_1)$ , and Bob has an RSA system with public key  $(n_2, b_2)$  and private key  $(a_2, p_2, q_2)$ . Alice wants to send Bob a message  $x$  using these systems to encrypt and authenticate. Indicate what should Alice do and what values should Alice send to Bob through the internet.

**Problem 6.**

Explain why birthday attack can be used to attack a hash function, but not suitable to use for attacking an public key encryption function.

## Set C

### Problem 7.

Alice suggested a simple MAC function to Bob as follows. Suppose Alice and Bob share a secret key  $K$  with length of at least 128 bits. For a message  $x$  of any length greater than 128 bits, define the MAC value as:

$$MAC_K(x) = x \bmod K.$$

Do you think this is a good MAC function? Explain why.

**Problem 8.**

Define a public key signature scheme as follows. Let

$$\mathcal{K} = \{(p, \alpha, a, \beta) : \beta \equiv \alpha^a \pmod{p}\}.$$

where  $p$  is a large prime and  $\alpha$  is a primitive number of  $\mathbb{Z}_p$ . The values of  $p, \alpha$  and  $\beta$  are public and  $a$  is secret.

For  $K = (p, \alpha, a, \beta) \in \mathcal{K}$  and for a secret random number  $k, 1 \leq k \leq p - 1$ , the signature of a message  $x \in \mathbb{Z}_p^*$  is

$$\text{sig}_K(x, k) = (\gamma, \delta),$$

where

$$\gamma = (\alpha^k \pmod{p})$$

and

$$\delta = (x - k\gamma)a^{-1} \pmod{p - 1}.$$

Write down the verification algorithm for this signature scheme.

**Problem 9.**

Suppose Bob has an RSA public key system with the public key certificated. Both Alice and Bob have SHA-256 and AES algorithms. If Alice wants to communicate with Bob, then what should Alice do to let the communication efficient, secure, and authenticated?