CS 5400 Graduate Seminar
Technical Report

MGDD with block size 4 and its application to sampling designs

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Contents

Abstract

In this report we give a complete solution of the existence of modified group divisible designs with block size 4. Then we give an application of the design to some interesting sampling designs.

This is a sample report for the purpose of LaTeXtemplate. So most content is omitted.

Chapter 1

Introduction

In this chapter, we give the background and overview of this topic.

. . . .

A *group divisible design* (GDD) is a triple $(X, \mathcal{G}, \mathcal{B})$ which satisfies the following properties:

1. $\mathcal{G}$ is a partition of a set X (of points) into subsets called *groups*,

2. $\mathcal{B}$ is a family of subsets of X (called *blocks*) such that a group and a block contain at most one common point,

3. every pair of points from distinct groups occurs in exactly λ blocks.

A *modified group divisible design* (MGDD) was first introduced in [1].

Definition 1.1 *Let X be a set of mn points where the points of X are denoted as $(x_i, y_j), 0 \leq i \leq m-1, 0 \leq j \leq n-1$. Let $\mathcal{B}$ be a collection of subsets of X (called blocks), which satisfies the following conditions:*

1. *$|B| = k$ for every block $B \in \mathcal{B}$;*

2. *every pair of points (x_{i_1}, y_{j_1}) and (x_{i_2}, y_{j_2}) of X are contained in exactly λ blocks, where $i_1 \neq i_2$ and $j_1 \neq j_2$.*

3. *the pair of points (x_{i_1}, y_{j_1}) and (x_{i_2}, y_{j_2}) with $i_1 = i_2$ or $j_1 = j_2$ are not contained in any blocks.*

Then we call $(X, \mathcal{B})$ *a modified group divisible design and denote it by* $MGD[k, \lambda, m, mn]$. *The subsets* $\{(x_i, y_j)|\ 0 \le i \le m-1\}$, *where* $0 \le j \le n-1$ *are called* groups *and the subsets* $\{(x_i, y_j)|\ 0 \le j \le n-1\}$, *where* $0 \le i \le m-1$ *are called* holes.

Modified group divisible designs are motivated by the existence problem of resolvable group divisible designs. An MGDD can be seen as a GDD with holes.

Chapter 2

Related work

2.1 Lemmas and Theorems

By simple calculation, we can easily obtain the following necessary conditions for the existence of an MGDD.

Lemma 2.1 . *The necessary conditions for the existence of an* $MGD[k, \lambda, m, nm]$ *are that* $m \geq k, n \geq k, \lambda(n-1)(m-1) \equiv 0 \pmod{k-1}$ *and* $\lambda nm(n-1)(m-1) \equiv 0 \pmod{k(k-1)}$.

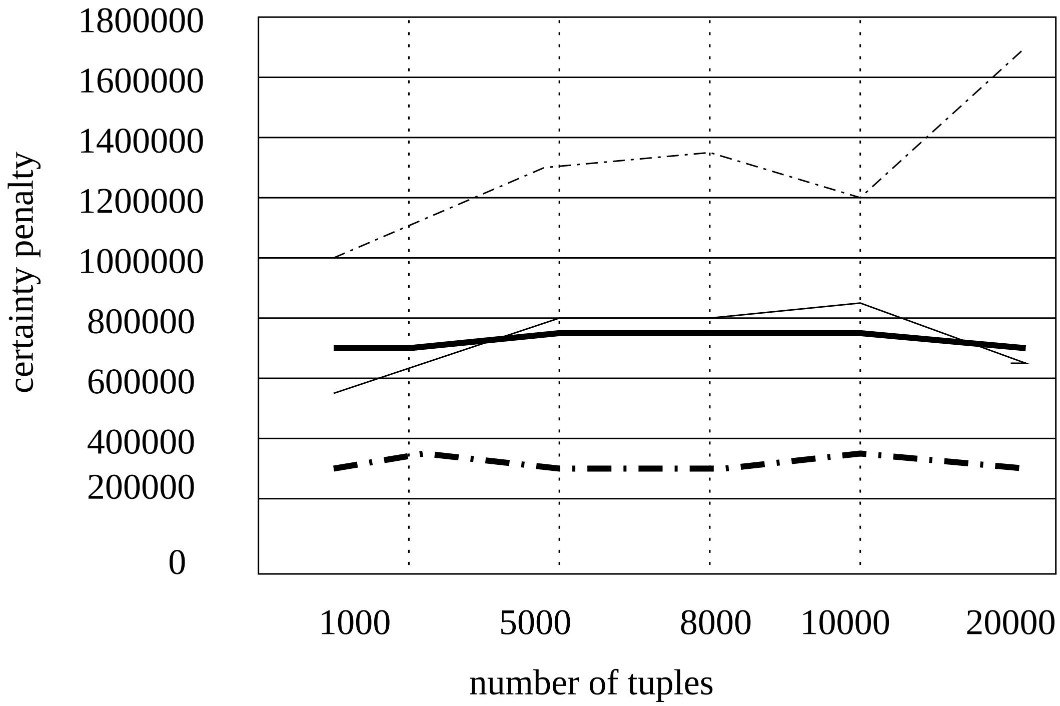
In [1] it is proved that the necessary conditions are sufficient when k = 3. However, when k = 4, these conditions are not sufficient. A counter-example is that an $MGD[4, 1, 6, 24]$ does not exist because there do not exist two mutually orthogonal Latin squares of order 6. The existence of MGDD with block size four was discussed in [2, 5]. The following theorem gives an almost complete solution.

Theorem 2.2 ([2, 5]) *An* $MGD[4, \lambda, m, nm]$ *exists whenever* $m, n \geq 4, \lambda(n-1)(m-1) \equiv 0 \pmod 3$, *except when* $\lambda = 1$ *and* $\{m, n\} = \{6, 4\}$, *and possibly when* $\lambda = 1$ *and* $\{m, n\} \in \{\{6, 16\}, \{6, 22\}, \{10, 15\}, \{10, 18\}\}$.

In this note, we will construct the four unknown designs directly and give the following complete solution for the existence of MGDD with block size 4.

Theorem 2.3 *An* $MGD[4, \lambda, m, nm]$ *exists if and only if* $m, n \geq 4$ *and* $\lambda(n-1)(m-1) \equiv 0 \pmod 3$, *except when* $\lambda = 1$ *and* $\{m, n\} = \{6, 4\}$.

2.2 Other conditions



Chapter 3

Constructions

In this section, we give the constructions of four MGDD. First we construct an $MGD[4, 1, 10, 10 \cdot 18]$. To do that, we consider an incomplete MGDD. Suppose that the points of one hole is deleted from an $MGD[4, 1, m, nm]$. Then the blocks can be divided into two parts: one part with blocks of size 4 and other part with blocks of size 3. Furthermore, all the blocks of size 3 can be partitioned into m partial parallel classes such that the points in the blocks of each class contain all the points of the MGDD except the points of one group and the missing hole.

3.1 An example

In the following, the notation "$+d$ mod g" denotes that all elements of the base blocks should be taken cyclically by adding d (mod g) to them, while the infinite point x, if it occurs in the base block, is always fixed.

$MGD[4, 1, 10, 10 \cdot 18]$ with a missing hole $\{x_i, 1 \leq i \leq 10\}$:

Point set: $Z_{170} \cup \{x_i, 1 \leq i \leq 10\}$

Groups:

Holes:

Base blocks: developed by (+2 mod 170)

$\{2, 21, 125, 144\}$;	$\{1, 2, 4, 96\}$;	$\{2, 27, 64, 148\}$;
$\{2, 93, 108, 115\}$;	$\{1, 32, 135, 144\}$;	$\{2, 17, 45, 103\}$;
$\{2, 15, 146, 150\}$;	$\{2, 38, 76, 149\}$;	$\{1, 28, 110, 115\}$;
$\{2, 71, 83, 116\}$;	$\{2, 14, 118, 119\}$;	$\{1, 53, 94, 117\}$;
$\{2, 31, 39, 55\}$;	$\{2, 10, 54, 151\}$;	$\{1, 33, 40, 138\}$;
$\{1, 43, 165, 167\}$;	$\{1, 116, 127, 162\}$;	$\{1, 63, 126, 140\}$;
$\{1, 26, 83, 97\}$;	$\{1, 27, 76, 108\}$;	$\{2, 5, 43, 154\}$

$\{9, 14, 81\}$;	$\{8, 107, 162\}$;	$\{13, 26, 105\}$;
		+10 mod 170 missing $0, 10, ...$
$\{14, 136, 142\}$;	$\{10, 93, 169\}$;	$\{5, 48, 157\}$.
		+10 mod 170 missing $1, 11, ...$

3.2 General constructions

Chapter 4

An application

We give an application of MGDD to some kind of sampling designs in this chapter.

Chapter 5

Recommendations

We can put future works or research plan, etc here.

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